Lecture 36

Robb T. Koether

Hampden-Sydney College

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- Dominance
- Simple Card Games
 - More Examples
 - The Travelers' Dilemma
 - The Diners' Dilemma
 - The Colonel Blotto Game
- Assignment

Outline

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Definition (Row Dominance)

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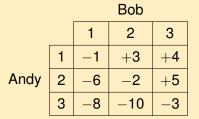
 In each case, the dominating row or column is the one that is more advantageous to that player.

- A dominated row can be removed from the game, because a rational row player would never choose it.
- A dominated column can be removed from the game, because a rational column player would never choose it.

- A dominated row can be removed from the game, because a rational row player would never choose it.
- A dominated column can be removed from the game, because a rational column player would never choose it.
- In this manner, larger games can often be reduced to smaller, simpler games.

Example with 3 Options

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 Our example of Andy and Bob choosing a number from 1 to 3 is a zero-sum game.

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		Bob		
		1	2	3
Andy	1	-1	+3	+4
	2	-6	-2	+5
	3	-8	-10	-3

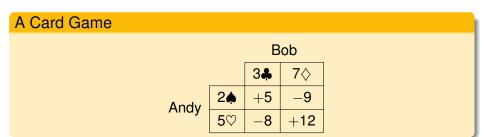
- Our example of Andy and Bob choosing a number from 1 to 3 is a zero-sum game.
- \bullet Through row and column dominance, we can reduce it to a trivial 1 \times 1 game.

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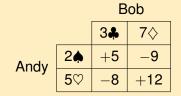


- Andy is holding the 2 of spades (2♠) and the 5 of hearts (5♡).
- Bob is holding the 3 of clubs (3♣) and the 7 of diamonds (7♦).
- Each player plays one of his cards.
- If the colors match, the Andy wins the sum of the numbers.
- If the colors do not match, Bob player wins the sum of the numbers.
- Write the payoff matrix.

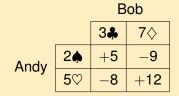


A Card Game Bob 3♣ 7♦ Andy 2♠ +5 -9 5♡ -8 +12

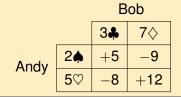
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- It is also clear that there is no row or column dominance.



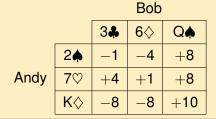
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- What should Andy and Bob do?



- It is clear that neither player has a pure strategy.
- It is also clear that there is no row or column dominance.
- What should Andy and Bob do?
- We will deal with that in the next lecture.

- Andy is holding the 2 of spades (2♠), the 7 of hearts (7♥), and the king of diamonds (K♦).
- Bob is holding the 3 of clubs (3♣), the 6 of diamonds (6♦), and the queen of spades (Q♠).
- Each player plays one of his cards.
 - If both are face cards (queen or king), then Andy wins \$10.
 - If only one is a face card, then the one who played the face card wins \$8.
 - If neither is a face card, then whichever player played the larger number, wins the difference between the numbers, in dollars.

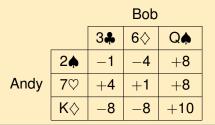
Another Card Game



Another Card Game Bob 6♦ 3. Q 2 -4+8Andy 7♡ +1+4+8K♦ -8-8+10

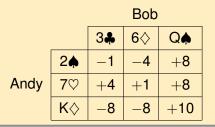
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Another Card Game



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- Use row and column dominance to reduce the game as far as possible.

Another Card Game



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- What will Andy and Bob do?

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- At their destination, they discovered that the airline lost their suitcases.
- The suitcases, as well as their contents, were identical, and therefore of the same value.
- The airline will reimburse them the value of their suitcases and contents up to a maximum of \$5, with a minimum reimbursement of \$2.
- The airline manager separates Andy and Bob and asks each to write down the value of his suitcase and its contents.

The Travelers' Dilemma

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- Furthermore, the one who wrote down the smaller value receives the smaller value **plus** \$2...
- While the one who wrote down the larger value receives the smaller value minus \$2.
- What values should Andy and Bob write down in order to receive as much as they can from the airline?

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- What is their optimal strategy?

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 - At least one regiment must be sent to each battlefield.
- Col. Blotto's opponent, Col. Lotso, is facing the same decision.
- Neither Col. Blotto nor Col. Lotso knows the other's strategy.
- How should Col. Blotto divide his army?

The Greatly Simplified Colonel Blotto Game

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- What numbers should Andy and Bob write down?

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• Work the problems on Handout #3.