Dominance

Lecture 36

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Outline

Dominance

Definition (Row Dominance)

In a payoff matrix, row *r* dominates row *s* if every payoff in row *r* is greater than or equal to the *corresponding* payoff in row *s*.

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Definition (Column Dominance)

In a payoff matrix, column *c* dominates column *d* if every payoff in column *c* is less than or equal to the *corresponding* payoff in column *d*.

Dominance

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Definition (Column Dominance)

In a payoff matrix, column *c* dominates column *d* if every payoff in column *c* is less than or equal to the *corresponding* payoff in column *d*.

- In each case, the dominating row or column is the one that is more advantageous to that player.

Dominance

- A dominated row can be removed from the game, because a rational row player would never choose it.
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- A dominated row can be removed from the game, because a rational row player would never choose it.
- A dominated column can be removed from the game, because a rational column player would never choose it.
- In this manner, larger games can often be reduced to smaller, simpler games.

Example with 3 Options

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		Bob		
		1	2	3
Andy	1	-1	$+3$	$+4$
	2	-6	-2	$+5$
	3	-8	-10	-3

- Our example of Andy and Bob choosing a number from 1 to 3 is a zero-sum game.

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- Our example of Andy and Bob choosing a number from 1 to 3 is a zero-sum game.
- Through row and column dominance, we can reduce it to a trivial 1×1 game.

Outline

A Card Game

A Card Game

- Andy is holding the 2 of spades (2♠) and the 5 of hearts (5♡).
- Bob is holding the 3 of clubs (3♣) and the 7 of diamonds (7♢).
- Each player plays one of his cards.
- If the colors match, the Andy wins the sum of the numbers.
- If the colors do not match, Bob player wins the sum of the numbers.
- Write the payoff matrix.

A Card Game

		Bob	
		$3\clubsuit$	$7\diamondsuit$
Andy	$2\spadesuit$	$+5$	-9
	$5\heartsuit$	-8	$+12$

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- It is also clear that there is no row or column dominance.

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- It is also clear that there is no row or column dominance.
- What should Andy and Bob do?

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- It is also clear that there is no row or column dominance.
- What should Andy and Bob do?
- We will deal with that in the next lecture.

A Card Game

A Card Game

- Andy is holding the 2 of spades (2♠), the 7 of hearts (7♡), and the king of diamonds (K♢).
- Bob is holding the 3 of clubs (3♣), the 6 of diamonds (6♢), and the queen of spades (Q♠).
- Each player plays one of his cards.
  - If both are face cards (queen or king), then Andy wins $10.
  - If only one is a face card, then the one who played the face card wins $8.
  - If neither is a face card, then whichever player played the larger number, wins the difference between the numbers, in dollars.

Another Card Game

Another Card Game

		Bob		
		3♣	6♢	Q♠
Andy	2♠	−1	−4	+8
	7♡	+4	+1	+8
	K♢	−8	−8	+10

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- Use row and column dominance to reduce the game as far as possible.

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- Does either player have a pure strategy?
- Use row and column dominance to reduce the game as far as possible.
- What will Andy and Bob do?

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- At their destination, they discovered that the airline lost their suitcases.
- The suitcases, as well as their contents, were identical, and therefore of the same value.
- The airline will reimburse them the value of their suitcases and contents up to a maximum of \$5, with a minimum reimbursement of \$2.
- The airline manager separates Andy and Bob and asks each to write down the value of his suitcase and its contents.

The Travelers' Dilemma

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- Furthermore, the one who wrote down the smaller value receives the smaller value **plus** $2...

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- Furthermore, the one who wrote down the smaller value receives the smaller value **plus** $2...
- While the one who wrote down the larger value receives the smaller value **minus** $2.
- What values should Andy and Bob write down in order to receive as much as they can from the airline?

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- If Andy or Bob were dining alone, they would order the $10 meal because they feel the $20 is overpriced.
- But each figures that if they order the $20 meal and the other orders the $10 meal, then he will get the $20 meal for $15 dollars!

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- If Andy or Bob were dining alone, they would order the $10 meal because they feel the $20 is overpriced.
- But each figures that if they order the $20 meal and the other orders the $10 meal, then he will get the $20 meal for $15 dollars!
- What is their optimal strategy?

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  - At least one regiment must be sent to each battlefield.

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  - On each battlefield, the army with the greater number of regiments will win.
  - At least one regiment must be sent to each battlefield.
- Col. Blotto's opponent, Col. Lotso, is facing the same decision.
- Neither Col. Blotto nor Col. Lotso knows the other's strategy.
- How should Col. Blotto divide his army?

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- In each position of their lists, the larger number wins.
- The payoff is the net number of wins for the players.
- What numbers should Andy and Bob write down?

Outline

Assignment

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- Work the problems on Handout #3.